1. Validity. Make up an argument with the described premises and conclusion, or say why such an argument is impossible. (10 pts each)

(a) Invalid, with two true premises, and a false conclusion.

Donald Trump is president. Every president has been a man. Therefore, Donald Trump got more votes than Hillary Clinton.

(b) Valid, with one true premise, and a false conclusion.

Impossible, because a valid argument cannot have true premises and a false conclusion.

(c) Invalid, with two true premises, and a true conclusion.

Rats have four legs. Cats have four legs. Therefore, dogs have four legs.

2. Translations (10 pts each)

(a) Translate the following sentences from English into the formal language of Tarski’s World.

i. c is a cube, and it’s large if a is.

\[ \text{Cube}(c) \land (\text{Large}(a) \rightarrow \text{Large}(c)) \]

ii. Either a or b is small, and one is a tetrahedron if and only if the other is.

\[ (\text{Small}(a) \lor \text{Small}(b)) \land (\text{Tet}(a) \leftrightarrow \text{Tet}(b)) \]

iii. If a is a medium dodecahedron, then neither c nor d is.

\[ (\text{Medium}(a) \land \text{Dodec}(a)) \rightarrow \neg((\text{Medium}(c) \land \text{Dodec}(c)) \lor (\text{Medium}(d) \land \text{Dodec}(d))) \]
(b) Give ordinary English translations of the following sentences in the formal language of Tarski's World.

i. Small(a)↔¬Cube(a)

a is small if and only if it is not a cube.

ii. (Cube(a)∨Tet(a))→Large(a)

If a is either a cube or a tet then it is large.

iii. (Small(b)∧Dodec(b))∧(Dodec(c)→Small(c))

b is a small dodecahedron, and c is a dodecahedron only if it's small too.

3. Complete the following two incomplete proofs. Fill in the rule used on each line, and which prior lines it depends on. (20 pts each)

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4. If the probability that a is a tet given that it is small is 1/8, and the probability that it is a tet given that it is not small is 1/2, and the probability that it is small is .8, then what is the probability that it is small given that it’s a tet? (20 pts)

Since \( P(A \text{ given } B) = \frac{P(A \land B)}{P(B)} \), it is also true that \( P(A \land B) = P(A \text{ given } B) \times P(B) \). Thus, we can use the given information to calculate lines 1 and 3 of the table. (The other two lines get the amount that is left over.)

\[
P(\text{Small}(a) \land \text{Tet}(a)) = \frac{.1}{.8 + .1} = \frac{1}{9}
\]

\[
P(\text{Tet}(a)) = \frac{1}{4}
\]

Thus, the probability that a is small given that it’s a tet is 1/2.

(Note: I used two different methods for the probability question on the two samples, but they are both ways of getting the same information, and either method will work for either problem.)